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A proposal for covariant renormalizable field theory of gravity

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The class of covariant gravity theories which have nice ultraviolet behavior and seem to be (super)-renormalizable is proposed. The apparent breaking of Lorentz invariance occurs due to the coupling with the effective fluid which is induced by Lagrange multiplier constrained scalar field. Spatially-flat FRW cosmology for such covariant field gravity may have accelerating solutions. Renormalizable versions of more complicated modified gravity which depends on Riemann and Ricci tensor squared may be constructed in the same way.

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Introduction. The main problem related with the quantization of the gravity is that if we consider the perturbations from the flat background, which has a Lorentz invariance, by using General Relativity, there appear the non-renormalizable divergences from the ultraviolet region in momentum space. Higher-derivative gravity may be renormalizable (see book [1]) but the well-known unitarity problem cannot be solved there. The idea proposed in ref.[2] (for cosmological applications, see [3]) is to modify the ultraviolet behavior of the graviton propagator in Lorentz non-invariant way as $1/\left|\boldsymbol{k}\right|^{2z}$, where $\boldsymbol{k}$ is the spatial momenta and z could be 2, 3 or larger integers. They are defined by the scaling properties of space-time coordinates $(\boldsymbol{x},t)$ as follows, $\boldsymbol{x}\to b\boldsymbol{x}$, $t\to b^z t$. When $z=3$, the theory seems to be UV renormalizable. Then in order to realize the Lorentz non-invariance, one introduces the terms breaking the Lorentz invariance explicitly (or more precisely, breaking full diffeomorphism invariance) by treating the temporal coordinate and the spatial coordinates in a different way.

Such model has invariance under time reparametrization and time dependent spatial diffeomorphisms: $\delta x^i = \zeta^i(t,\boldsymbol{x})$, $\delta t = f(t)$. Here $\zeta^i(t,\boldsymbol{x})$ and $f(t)$ are arbitrary functions.

In ref.[5], Hořava-like gravity model with full diffeomorphism invariance has been proposed. When we consider the perturbations from the flat background, which has Lorentz invariance, the Lorentz invariance of the propagator is dynamically broken by the non-standard coupling with a perfect fluid. The obtained propagator behaves as $1/\boldsymbol{k}^{2z}$ with $z=2,3,\cdots$ in the ultraviolet region and the model could be perturbatively power counting (super-)renormalizable if $z\geq 3$. The price for such covariant renormalizability is the presence of unknown (string-inspired?) fluid. This fluid could not correspond to the usual fluid like, radiation, baryons, dust, etc. The model can be consistently constructed when the equation of state (EoS) parameter $w\neq -1\,,1/3$. For usual particles in the high energy region, the corresponding fluid is relativistic radiation for which $w\to 1/3$. We need the non-relativistic fluid even in the high energy region.

Recently dust fluid with $w=0$ has been constructed for the scalar theory by introducing the Lagrange multiplier field, which gives a constraint on the first scalar field [4]. In this letter, we construct a fluid with arbitrary constant w from the scalar field which satisfies a constraint. Due to the constraint, the scalar field is not dynamical and even in the high energy region, one can obtain a non-relativistic fluid. By the coupling with the fluid, one can get the full diffeomorphism invariant Lagrangian (actually, class of such gravitational Lagrangians) given completely in terms of fields variables. It is demonstrated that such theory has the good properties of Lorentz non-invariant gravity (its conjectured renormalizability) being on the same time the covariant one. Moreover, in simplest case its spatially-flat FRW cosmology may have accelerating solutions.

Review of covariant renormalizable gravity. Let us briefly review the covariant renormalizable gravity of ref.[5]. The starting action is

$$S=\int d^4x\sqrt{-g}\left\{\frac{R}{2\kappa^2}-\alpha\left(T^{\mu\nu}R_{\mu\nu}+\beta TR\right)^2\right\}\ . \tag{1}$$

Here $T_{\mu\nu}$ is energy-momentum tensor of the exotic fluid. The action (1) is fully diffeomorphism invariant. We consider the perturbation from the flat background $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$. We now choose the following gauge conditions:

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$h_{tt} = h_{ti} = h_{it} = 0$. For the perfect fluid, the energy-momentum tensor in the flat background has the following form:

$$T_{tt} = \rho \ , \quad T_{ij} = p\delta_{ij} = w\rho\delta_{ij} \ . \tag{2}$$

Here w is the equation of state (EoS) parameter. Then one finds

$$\begin{aligned} & T^{\mu\nu}R_{\mu\nu} + \beta T R \\ & = \rho \left[\left\{ -\frac{1}{2} + \frac{w}{2} + \left(-1+3w\right)\beta \right\} \partial_t^2 \left(\delta^{ij}h_{ij}\right) + \left(w - \beta + 3w\beta\right)\partial^i\partial^j h_{ij} + \left(-w + \beta - 3w\beta\right)\partial_k\partial^k\left(\delta^{ij}h_{ij}\right) \right] \end{aligned} \tag{3}$$

If we choose

$$\beta = -\frac{w-1}{2\left(3w-1\right)} \ , \tag{4}$$

the second term in the action (1) becomes

$$\alpha\left(T^{\mu\nu}R_{\mu\nu} + \beta T R\right)^2 = \alpha\rho^2\left(\frac{w}{2} + \frac{1}{2}\right)^2 \left\{\partial^i\partial^j h_{ij} - \partial_k\partial^k\left(\delta^{ij}h_{ij}\right)\right\}^2 \ , \tag{5}$$

which does not contain the derivative with respect to t and breaks the Lorentz invariance. We now assume ρ is almost constant. Then in the ultraviolet region, where $\boldsymbol{k}$ is large, the second term in the action (1) gives the propagator behaving as $1/\left|\boldsymbol{k}\right|^4$, which renders the ultraviolet behavior (compared with Eq.(1.4) in [2]). Note that the form (4) indicates that the longitudinal mode does not propagate but only the transverse mode propagates.

There are two special cases in the choice of w: when $w = -1$, which corresponds to the cosmological constant, one gets $T^{\mu\nu}R_{\mu\nu} + \beta T R = 0$ and therefore we do not obtain $1/\boldsymbol{k}^4$ behavior. When $w = 1/3$, which corresponds to the radiation or conformal matter, β diverges and therefore there is no solution.

The apparent breakdown of the Lorentz symmetry in (5) occurs due to the coupling with the perfect fluid. The action (1) is invariant under the diffeomorphisms in four dimensions and the energy-momentum tensor $T_{\mu\nu}$ of the non-standard fluid in the action should transform as a tensor under the diffeomorphisms. The existence of the fluid, however, effectively breaks the Lorentz symmetry, which is the equivalence between the different inertial frames of reference. Note that the expression (2) is correct in the reference frame where the fluid does not flow, or the velocity of the fluid vanishes. In other reference frames, there appear non-vanishing $T_{it} = T_{ti}$ components and there could appear the derivative with respect to time, in general.

In the arguments after (1), the flat background is considered but the arguments could be generalized for the curved background: in the curved spacetime, due to the principle of the general relativity, we can always choose the local Lorentz frame. The local Lorentz frame has (local) Lorentz symmetry. Even in the local Lorentz frame, the perfect fluid might flow and $T_{it} = T_{ti}$ components might not vanish. By boosting the frame, which is the (local) Lorentz transformation, we have a special local Lorentz frame, where the fluid does not flow. In the Lorentz frame, one can use the above arguments and find there is no breakdown of the unitarity. Conversely, in a general coordinate frame, $T^{\mu\nu}R_{\mu\nu} + \beta T R$ can have a derivative with respect to time.

The action (1) gives $z = 2$ theory. In order that the theory could be ultra-violet power counting renormalizable in $3+1$ dimensions, $z = 3$ theory is necessary. In order to obtain such a theory, we note that, for any scalar quantity Φ, if we choose

$$\gamma = \frac{1}{3w-1} \ , \tag{6}$$

one obtains

$$T^{\mu\nu}\nabla_\mu\nabla_\nu\Phi + \gamma T\nabla^\rho\nabla_\rho\Phi = \rho\left(w+1\right)\partial_k\partial^k\Phi \ , \tag{7}$$

which does not contain the derivative with respect to time coordinate t. This is true even if the coordinate frame is not local Lorentz frame. The derivative with respect to time coordinate t is not contained in any coordinate frame, where the perfect fluid does not flow. Then if we consider

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{2\kappa^2} - \alpha\left(T^{\mu\nu}R_{\mu\nu} + \beta T R\right)\left(T^{\mu\nu}\nabla_\mu\nabla_\nu + \gamma T\nabla^\rho\nabla_\rho\right)\left(T^{\mu\nu}R_{\mu\nu} + \beta T R\right)\right\} \ , \tag{8}$$

with $\beta = -\frac{w-1}{2(3w-1)}$ and $\gamma = \frac{1}{3w-1}$, we obtain $z = 3$ theory, which seems to be renormalizable. In general, for the case

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - \alpha\left\{\left(T^{\mu\nu}\nabla_\mu\nabla_\nu + \gamma T\nabla^\rho\nabla_\rho\right)^n\left(T^{\mu\nu}R_{\mu\nu} + \beta T R\right)\right\}^2\right] \ , \tag{9}$$

with a constant n, we obtain $z = 2n+2$ theory which is super-renormalizable for $n \geq 1$. Usually n should be an integer but in general, we may consider pseudo-local differential operator $(T^{\mu\nu}\nabla_\mu\nabla_\nu + \gamma T \nabla^\rho \nabla_\rho)^n$ with non-integer n (e.g. $n = 1/2, 3/2$ etc.). We should also note that there are special cases, that is, $w = -1$ and $w = 1/3$.

The second terms in the actions (1), (8), (9), and (9), which effectively break the Lorentz symmetry, are relevant only in the high energy/UV region since they contain higher derivative terms. In the IR region, these terms do not dominate and the usual Einstein gravity follows as a limit.

In [8, 9], $F(R)$-gravity [6] version of the Hořava-Lifshitz-like gravity has been proposed. The renormaliability of this class of $F(R)$-gravity has been established in [10]. In [10], the covariant version of the Hořava-Lifshitz-like $F(R)$-gravity has been also presented. Its action is given by

$$
\begin{aligned}
S_{F(\tilde{R}_{\rm cov})} &= \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}F(R_{\rm cov})\,, \\
R_{\rm cov} &= \begin{cases} R - 2\alpha\kappa^2\left(T^{\mu\nu}R_{\mu\nu} + \beta T R\right)^2\,, & z=2\,, \\ R - 2\kappa^2\alpha\left(T^{\mu\nu}R_{\mu\nu} + \beta T R\right)\left(T^{\mu\nu}\nabla_\mu\nabla_\nu + \gamma T\nabla^\rho\nabla_\rho\right)\left(T^{\mu\nu}R_{\mu\nu} + \beta T R\right) & z=3\,, \\ R - 2\kappa^2\alpha\left\{\left(T^{\mu\nu}\nabla_\mu\nabla_\nu + \gamma T\nabla^\rho\nabla_\rho\right)^n\left(T^{\mu\nu}R_{\mu\nu} + \beta T R\right)\right\}^2 & z = 2n+2\,. \end{cases}
\end{aligned} \tag{10}
$$

Although it has not been shown, the theories (10) are expected to be renormalizable when $z \geq 3$, which may be demonstrated using the same arguments as in ref. [5] or in this section.

Covariant renormalizable field theory of gravity with Lagrange multiplier. In this section, the covariant gravity coupled with the (Lagrange multiplier induced)fluid is constructed.

We consider the following constrained action for the scalar field ϕ

$$S_\phi = \int d^4x\sqrt{-g}\left\{-\lambda\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + U(\phi)\right)\right\}\,. \tag{11}$$

Here λ is the Lagrange multiplier field, which gives a constraint

$$\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + U(\phi) = 0\,, \tag{12}$$

that is, the vector $(\partial_\mu\phi)$ is time-like. At least locally, one can choose the direction of time to be parallel to $(\partial_\mu\phi)$. Then Eq. (12) has the following form:

$$\frac{1}{2}\left(\frac{d\phi}{dt}\right)^2 = U(\phi)\,. \tag{13}$$

The equation given by the variation of ϕ will be discussed later.

We now *define* a tensor $T^\phi_{\mu\nu}$ corresponding to the energy momentum tensor of the scalar field with a potential $V(\phi)$:

$$T^\phi_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\rho\phi\partial^\rho\phi + V(\phi)\right)\,. \tag{14}$$

The "energy density" ρ_ϕ and "pressure" p_ϕ become:

$$p_\phi = \frac{1}{2}\left(\frac{d\phi}{dt}\right)^2 - V(\phi) = U(\phi) - V(\phi)\,, \quad \rho_\phi = \frac{1}{2}\left(\frac{d\phi}{dt}\right)^2 + V(\phi) = U(\phi) + V(\phi)\,. \tag{15}$$

Here, the constraint (13) is used. We should note that $V(\phi)$ is not identical with $U(\phi)$: $V(\phi) \neq U(\phi)$. In case $V(\phi) = U(\phi)$, Eq.(15) tells that $p_\phi = 0$, which corresponds to dust with $w_\phi \equiv p_\phi/\rho_\phi = 0$. Note that quantization of constrained theories is quite non-trivial task (see reviews[7]).

For simplicity, we choose $V(\phi)$ and $U(\phi)$ to be constants:

$$U(\phi) = U_0\,, \quad V(\phi) = V_0\,. \tag{16}$$

Then if $U_0 = V_0$, the EoS parameter w_ϕ vanish. In general case, one has $w_\phi = \frac{U_0 - V_0}{U_0 + V_0}$. Let us now use $T^\phi_{\mu\nu}$ as a energy-momentum tensor in the previous section. Since (4) shows $\beta = -\frac{w-1}{2(3w-1)} = \frac{V_0}{2U_0 - 4V_0}$. One can simplify

$$T^{\phi\,\mu\nu}R_{\mu\nu} + \beta T^\phi R = \partial^\mu\phi\partial^\nu\phi R_{\mu\nu} + U_0 R\,. \tag{17}$$

Here, Eqs.(12), (14), and (16) are used. We also find γ in (6) has the following form: $\gamma = \frac{U_0 - V_0}{2U_0 - 4V_0}$, which gives, by using (7),

$$T^{\phi\,\mu\nu}\nabla_\mu\nabla_\nu\Phi + \gamma T^\phi \nabla^\rho\nabla_\rho\Phi = \partial^\mu\phi\partial^\nu\phi\nabla_\mu\phi\nabla_\nu\Phi + 2U_0\nabla^\rho\nabla_\rho\Phi\,. \tag{18}$$

Eq. (17) enables to write down $z=2$ total action corresponding to (1) as

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{2\kappa^2} - \alpha\left(\partial^\mu\phi\partial^\nu\phi R_{\mu\nu} + U_0 R\right)^2 - \lambda\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + U_0\right)\right\}\,. \tag{19}$$

On the other hand, $z=3$ total action corresponding to (8) has the following form:

$$\begin{aligned} S &= \int d^4x\sqrt{-g}\left\{\frac{R}{2\kappa^2} - \alpha\left(\partial^\mu\phi\partial^\nu\phi R_{\mu\nu} + U_0 R\right)\left(\partial^\mu\phi\partial^\nu\phi\nabla_\mu\nabla_\nu + 2U_0\nabla^\rho\nabla_\rho\right)\left(\partial^\mu\phi\partial^\nu\phi R_{\mu\nu} + U_0 R\right)\right. \\ &\quad \left. -\lambda\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + U_0\right)\right\}\,, \end{aligned} \tag{20}$$

and $z = 2n+2$ action is given by

$$\begin{aligned} S &= \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - \alpha\left\{\left(\partial^\mu\phi\partial^\nu\phi\nabla_\mu\nabla_\nu + 2U_0\nabla^\rho\nabla_\rho\right)^n\left(\partial^\mu\phi\partial^\nu\phi R_{\mu\nu} + U_0 R\right)\right\}^2\right. \\ &\quad \left. -\lambda\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + U_0\right)\right]\,, \end{aligned} \tag{21}$$

Note that the actions (19), (20), and (21) are totally diffeomorphism invariant and only given in terms of the local fields. We also note that the actions (19), (20), and (19) do not depend on V_0.

By the variation over ϕ, for example, for $z=2$ case in (19),one finds

$$0 = 4\alpha\partial^\mu\left\{\partial^\nu\phi R_{\mu\nu}\left(\partial^\rho\phi\partial^\sigma\phi R_{\rho\sigma} + U_0 R\right)\right\} + \partial^\mu\left(\lambda\partial_\mu\phi\right)\,. \tag{22}$$

For $z=3$ case (20) or $z=2n+2$ case (21), rather complicated equations follow.

For the perturbation from the flat background $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, we find

$$\begin{aligned} \partial^\mu\phi\partial^\nu\phi R_{\mu\nu} + U_0 R &= U_0\left\{\partial^i\partial^j h_{ij} - \partial_k\partial^k\left(\delta^{ij}h_{ij}\right)\right\}\,, \\ \partial^\mu\phi\partial^\nu\phi\nabla_\mu\nabla_\nu + 2U_0\nabla^\rho\nabla_\rho &= 2U_0\partial_k\partial^k\,. \end{aligned} \tag{23}$$

Then in the ultraviolet region, where $\boldsymbol{k}$ is large, the propagator behaves as $1/\left|\boldsymbol{k}\right|^4$ for $z=2$ case in (19) and therefore the ultraviolet behavior is rendered. In $z=3$ case in (20), the propagator behaves as $1/\left|\boldsymbol{k}\right|^6$ and therefore the model becomes renormalizable. In $z=2n+2$ case in (21), when $n\geq 1$, the model becomes super-renormalizable.

The $F(R)$-gravity corresponding to (10) is given by

$$\begin{aligned} S_{F(\tilde R_{\mathrm{cov}})} &= \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left\{F(R_{\mathrm{cov}}) - \lambda\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + U_0\right)\right\}\,, \\ R_{\mathrm{cov}} &= \left\{\begin{array}{ll} R - 2\alpha\kappa^2\left(\partial^\mu\phi\partial^\nu\phi R_{\mu\nu} + U_0 R\right)^2\,, & z=2\,, \\ R - 2\kappa^2\alpha\left(\partial^\mu\phi\partial^\nu\phi R_{\mu\nu} + U_0 R\right)\left(\partial^\mu\phi\partial^\nu\phi\nabla_\mu\nabla_\nu + 2U_0\nabla^\rho\nabla_\rho\right)\left(\partial^\mu\phi\partial^\nu\phi R_{\mu\nu} + U_0 R\right) & z=3\,, \\ R - 2\kappa^2\alpha\left\{\left(\partial^\mu\phi\partial^\nu\phi\nabla_\mu\nabla_\nu + 2U_0\nabla^\rho\nabla_\rho\right)^n\left(\partial^\mu\phi\partial^\nu\phi R_{\mu\nu} + U_0 R\right)\right\}^2 & z=2n+2\,. \end{array}\right. \end{aligned} \tag{24}$$

The action (24) is also totally diffeomorphism invariant and only given in terms of the local fields.

Discussion: cosmological applications. Let us make several remarks about FRW cosmology in the presence of matter. In order to obtain the FRW equations, we assume the following form of the metric: $ds^2 = -\mathrm{e}^{2b(t)}dt^2 + a(t)^2\sum_{i=1,2,3}\left(dx^i\right)^2$, and that the scalar field ϕ only depends on time. Then

$$\partial^\mu\phi\partial^\nu\phi R_{\mu\nu} + U_0 R = 6H^2U_0\mathrm{e}^{-2b}\,,\quad \partial^\mu\phi\partial^\nu\phi\nabla_\mu\nabla_\nu + 2U_0\nabla^\rho\nabla_\rho = -6U_0\mathrm{e}^{-2b}H\partial_t\,. \tag{25}$$

For simplicity, we consider the model with $z=2$ (19), then the action (19) has the following form:

$$S = \int d^4x a^3\left[\frac{\mathrm{e}^{-b}}{2\kappa^2}\left(6\dot H + 12H^2 - 6\dot bH\right) - 36\alpha\,U_0^2\mathrm{e}^{-3b}H^4 - \lambda\left(-\frac{\mathrm{e}^{-b}}{2}\dot\phi^2 + \mathrm{e}^bU_0\right)\right]\,. \tag{26}$$

The equation corresponding to the first FRW equation can be obtained by putting $b = 0$ after the variation over b and it has the following form:

$$\frac{3}{\kappa^2} H^2 = -108\alpha U_0^2 H^4 + 2\lambda U_0 + \rho_{\mathrm{matter}} \,. \tag{27}$$

Here we have used a constraint (13), which is valid even in the FRW universe and the usual matter energy-density ρ_{matter} is included. On the other hand, by considering the variation over a and putting $b = 0$, one obtains the equation corresponding to the second FRW equation for (20):

$$-\frac{1}{\kappa^2}\left(2\dot{H} + 3H^2\right) = 36\alpha U_0^2 \left(3H^4 + 4H^2\dot{H}\right) + p_{\mathrm{matter}} \,. \tag{28}$$

Here we have used a constraint (13), again and p_{matter} is the usual matter pressure. Note that Eq. (28) does not contain λ. By solving (28), $H = H(t)$. After that, by substituting the solution $H(t)$, we can find the form of λ.

In the early universe with large curvature, the contribution from the Einstein term, which corresponds to the right-hand sides in (27) and (28) is large, and the matter contribution could be neglected. Then a solution of (28) is given by

$$H = \frac{4}{3t} \,, \tag{29}$$

which expresses the (power law) accelerating expansion of the universe corresponding to the perfect fluid with $w = -1/2$. Then Eq.(27) gives

$$\lambda = \frac{32\alpha U_0}{3t^4} \,. \tag{30}$$

This accelerated FRW cosmology may be proposed to describe (quintessencial) inflationary era.

One may confirm that the actions (19), (20), and (21) admit a solution where $R = R_{\mu\nu} = 0$, which corresponds to the flat, Schwarzschild, or Kerr space-time, by investigating the Einstein equation:

$$0 = \frac{1}{2\kappa^2}\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right) + G_{\mu\nu}^{\mathrm{higher}} - \frac{\lambda}{2}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2} g_{\mu\nu}\left(\frac{1}{2}\partial_\rho\phi\partial^\rho\phi + U_0\right) \,. \tag{31}$$

Here $G_{\mu\nu}^{\mathrm{higher}}$ comes from the higher derivative term (the second term) in the action. When $R = R_{\mu\nu} = 0$, then $G_{\mu\nu}^{\mathrm{higher}} = 0$, by using the constraint equation (12), we find that Eq.(31) reduces to

$$0 = \lambda\partial_\mu\phi\partial_\nu\phi \,, \tag{32}$$

whose solution is $\lambda = 0$. Then the actions (19), (20), and (21) admit the solution with $R = R_{\mu\nu} = 0$, which includes the Schwarzschild solution

$$ds^2 = -\left(1 - \frac{M}{r}\right) dt^2 + \left(1 - \frac{M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \,. \tag{33}$$

In the Hořava gravity and the theories which we are considering, the dispersion relation of the graviton is given by

$$\omega = c_0 k^z \,, \tag{34}$$

in the high energy region. Here c_0 is a constant, ω is the angular frequency corresponding to the energy and k is the wave number corresponding to momentum. Then the phase speed v_{p} and the group speed v_{g} are given by

$$v_{\mathrm{p}} = \frac{\omega}{k} = c_0 k^{z-1} \,, \quad v_{\mathrm{g}} = \frac{d\omega}{dk} = c_0 z k^{z-1} \,, \tag{35}$$

which becomes larger and larger when k becomes larger and goes beyond the light speed. This shows that even in (33), the high energy graviton can escape from the horizon. Note that the horizon is null surface and therefore in the usual Einstein gravity, particle cannot escape from the horizon since the speed of the particle is always less than or equal to the light speed. In our model, however, the speed of the graviton can exceed the light speed and escape from the horizon. This indicates that some properties of black holes in the theory under consideration are similar to the ones of Hořava gravity.

As a remark, instead of (24), one may investigate the $F(R)$ type model where the action is given by

$$S_{R+F(\tilde{R}_{\rm cov})} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left\{\frac{R}{2\kappa^2} + F(R_{\rm cov}) - \lambda\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + U_0\right)\right\}\,. \tag{36}$$

Here R is the usual scalar curvature. Note, however, the model (36) is not always renormalizable. Such examples are $F(R_{\rm cov}) \propto R_{\rm cov}^n$ ($n \geq 3$). In order to investigate the renormalizability, let us consider the fluctuation from the flat background. Then $R_{\rm cov}^n$ ($n \geq 3$) term does not contain the second power of the fluctuation but only contains n-th or higher power terms like h^m ($n \geq m$). Then the $R_{\rm cov}^n$ term does not give any contribution to the propagator and therefore the ultraviolet structure of the divergence is never improved and there appear non-renormalizable divergences.

Another remark is about the emergence of the standard Newton law. In the original Hořava model [2], the lapse function N is restricted to only depend on the time coordinate, which is called "projectability condition". This condition could be natural since the original Hořava model has not full diffeomorphism invariance but the invariance under time reparametrization and time dependent spatial diffeomorphisms. As pointed out in [11], by imposing the projectability condition, the Newton law could not be reproduced. Even in the Hořava model, if the projectability condition is not imposed, the Newton law could be realized. The model proposed in this paper, however, has the full diffeomorphism invariance and therefore we need not to impose the the projectability condition. In the models (19), (20), and (21), the corrections to the Einstein gravity come from the second and third terms. The second term is the higher derivative term and therefore relevant only for the very short distance, which possibly corresponds to the Planck length. Then this term does not affect the Newton law which has not been checked for such a short distance. The third term only gives a constraint and irrelevant for the Newton law.

In summary, we proposed class of covariant gravity theories which have nice ultraviolet behavior and seem to be (super)-renormalizable in the same sense as Hořava gravity which is known to be Lorentz non-invariant. These covariant theories are coupled with some fluid which is induced by the corresponding Lagrange multiplier constrained scalar. The accelerating spatially-flat FRW unified cosmology may be constructed for $F(R)$ versions of such theory, which opens the bridge between modified gravity cosmology and renormalizability. Moreover, renormalizable versions of more complicated modified gravity may be constructed in the same way.

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